Roll No. _______ Code No. Q-4314

SECOND SEMESTER EXAMINATION 2021-22

M.Sc. MATHEMATICS

Paper - IV

Complex Analysis - II

Time : 3.00 Hrs. **Max. Marks : 80**

Total No. of Printed Page : 03 **Mini. Marks : 29**

Note: Question paper is divided into three sections. Attempt question of all three section as per direction. Distribution of Marks is given in each section.

Section - 'A'

Very short type question (in few words). **6x2=12**

Q.1 Attempt any six question from the following questions :

(i) Write Weiersfrass factorization theorem.

(ii) Define Analytic continuation along curve.

(iii) Write Mittag-Leftler theorem.

(iv) Define rank of an entire function.

(v) Write Poission inequality.

(vi) Write Borel's theorem.

(vii) Write Little Picard Theorem.

(viii) Write Green's Theorem.

(ix) Define Univalent functions.

Section - 'B'

Short answer question (In 200 words) **4x5=20**

Q.2 Attempt any four question from the following questions :

(i) If f(z) is an entire function and $f(o) \neq 0$ then f(z) = f(o) $e^{g(z)}$ P(z). Where g(z) is an entire function and P(z) is a product of primary factors.

(ii) Prove that $\lceil(z) = \int_{o}^{\infty} e^{-t}\, t^{z-1}\ dt\ \ (R_e z > o)$

(iii) Let U and V be two open subsets of C with $V \subset U$ and $\partial v \cap U = \phi$. If H is a component of U and $H \cap V \neq$ then $H \subset V$.

(iv) Show that when $o < b < l$, The Series

$\frac{1}{2}\log\left(1+b^2\right)+i\tan^{-1}b+\frac{z-ib}{1+ib}-\frac{1}{2}\frac{\left(z-ib\right)^2}{\left(1+ib\right)^2}+.........$ is analytic continuation of the function defined by the series $z=\frac{z^2}{2}+\frac{z^3}{3}-..........$

(v) Let $u\ :\ G \to R$ be a continuous function which has Mean Value Properly. Then u is harmonic.

(vi) If the real part of an entire function g(z) satisfies the inequality

$\text{Re}(g(z)) < r^{f+\varepsilon} \qquad \forall \varepsilon > 0$ and all sufficiently large r, then g(z) is a polynomial of degree not exceeding p.

(vii) State and prove Hadamard's Factorization theorem.

(viii) Let $F \varepsilon H(U-\{o\})$, F be one to one in U, F has a Pole of order I at z=0, with residue I and neither w_1 nor w_2 in F(U) then $|W_1 - W_2| \leq 4$.

Section - 'C'

Long answer/Essay type question. **4x12=48**

Q.3 Attempt any four question from the following questions :

(i) State and prove Weierstrass factorization theorem.

(ii) State and prove Runge's theorem.

(iii) State and prove Schwarz's Reflection principle.

(iv) Let $\gamma:[0,1]\rightarrow C$ be a path from a to b and let $\{(f_t, Dt): O\leq t\leq 1\}$ be an analytic continuation along There is a number (-) O such that if $\sigma:[0,1]\rightarrow C$ is any path from a to b with $|\gamma(t)-\sigma(t)|<\varepsilon$ for all t and if $\{(g_t, B_t):\ 0\leq t\leq l\}$ is any continuation along σ with $[go]_c=[fo]_a$ then $[g_1]_b=[f_1]_b$.

(v) Let $D = \{z : |z| < 1\}$ be the unit disc with the boundary $\partial O=\{z:|z|=1\}$ and let $f:\partial D\rightarrow R$ be a continuous function. Then there is a continuous function $u:D\rightarrow R$ such ment

(a) $u(z)=f(z)\ \forall z\varepsilon\partial D$.

(b) x is harmonic in D.

(vi) Let f(z) be an entire function of finite order f and $n(r)$ be the number of zeros of f(z) in the closed disc $|z|\leq r$, not including possible zeros at origin Then $n(r)\ =\ O(r^{\rho+\varepsilon})$ i.e.

$n(r)\ ,\ r^{\rho+\varepsilon}$ for large value of z.

(vii) State and prove Little Picard theorem.

(viii) State and prove Montel Cara theodory theorem.

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